# Fast Evaluating the Lightning Horizontal Electric Field Using an Adaptive Hermite Interpolation in Frequency Domain

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**The horizontal electric field of a lightning channel, $E_r$, is the excitation for analyzing the field-to-transmission line problem. The $E_r(t)$ in time domain can be obtained via the Fourier transform technique, which requires calculating the $E_r(\omega)$,expressed in the form of general Sommerfeld integral at different frequencies repeatedly. In order to reduce the computing time, in this paper, an adaptive Hermite strategy is proposed to fit the $E_r$ in a wide frequency range. The approach to calculate the $E_r$ and its derivatives with respect to frequency is presented in the first place, and the adaptive algorithm for construct the interpolation model is outlined as well. The numerical examples show that the method proposed in this paper is at least 5 times faster than the one using the even sampling approach.**

***Index Terms*—Adaptive algorithms, Electromagnetic transients, Interpolation.**

## I. Introduction

The horizontal electric field of a lightning channel, $E_r(\omega)$, acts as an excitation for analyzing the field-to-transmission line problem. The obstacles in evaluating the $E_r(\omega)$ can be summarized as follows. The $E_r(\omega)$ is expressed in the form of general Sommerfeld integral(GSI), which usually has a highly oscillatory and slowly damped integrand. To obtain the $E_r$ in time domain, it requires to evaluate the $E_r(\omega)$ at a number of frequencies repeatedly if the Fourier and its inverse technique(FFT-IFFT) is utilized, which results in a time consuming process.

## II. Development of the Algorithm

If the modified transmission line model with exponential decay (MTLE) is applied for the channel current, the formula of the $E_r(\omega)$ can be summarized as follows:

$$E_r(\omega) = E_{r,\mathrm{ideal}}(\omega) - \Delta E_r(\omega)\,, \quad (1)$$

$$\Delta E_r(\omega) = \frac{\mathrm{j}I_{\mathrm{base}}(\mathrm{j}\omega)}{2\pi\omega\varepsilon_0}\int_0^{+\infty}\frac{u_1}{(n^2u_0+u_1)}\lambda^2\,\mathrm{e}^{-u_0h}\,\frac{1-\mathrm{e}^{-(u_0+\gamma)H}}{u_0+\gamma}J_1(\lambda r)\mathrm{d}\lambda$$
$$= \frac{\mathrm{j}I_{\mathrm{base}}(\mathrm{j}\omega)}{2\pi\omega\varepsilon_0}\int_0^{+\infty}F(\omega,\lambda)J_1(\lambda r)\mathrm{d}\lambda = \frac{\mathrm{j}I_{\mathrm{base}}(\mathrm{j}\omega)}{2\pi\omega\varepsilon_0}S(\omega)\,, \quad (2)$$

where $E_{r,\mathrm{ideal}}(\omega)$ denotes the horizontal electric field assuming the ground is a perfectly conducting plane; $\gamma$, $\alpha$, and $v$ are three constants. $I_{\mathrm{base}}(\mathrm{j}\omega)$ is the Fourier transform of the lightning current at the channel base. $J_1$ is the first order Bessel function of the first kind. $k_1 = \sqrt{\omega^2\mu_0\varepsilon_r\varepsilon_0 + \mathrm{j}\omega\mu_0\sigma}$, $k_0 = \sqrt{\omega^2\mu_0\varepsilon_0}$, $n^2 = k_1^2/k_0^2$, $u_0 = \sqrt{\lambda^2 - k_0^2}$, and $u_1 = \sqrt{\lambda^2 - k_1^2}$.

### A. Evaluating $S(\omega)$ with Cubic Hermite Interpolation

The interval $[\omega_{\min}, \omega_{\max}]$ is divided into $N_{\mathrm{deri}}$ sub-intervals, and $S(\omega)$ can be interpolated by

$$S_{\mathrm{deri}}(\omega) = \sum_{n=1}^{N_{\mathrm{Deri}}} S_{\mathrm{deri},n}(\omega)[U(\omega-\omega_n) - U(\omega-\omega_{n+1})]\,, \quad (3)$$

where $U(\omega\text{-}\omega_n)$ is the step function with the shift $\omega_n$. $\omega_n$ and $\omega_{n+1}$ are the two endpoints of the $n$-th interval. The cubic Hermite polynomial in the $n$-th interval can be expressed as

$$S_{\mathrm{deri},n}(\omega) = \frac{3\Delta_n\xi^2 - 2\xi^3}{\Delta_n^{\ 3}}S_{n+1} + \frac{\Delta_n^{\ 3} - 3\Delta_n\xi^2 + 2\xi^3}{\Delta_n^{\ 3}}S_n$$
$$+\frac{\xi^2(\xi-\Delta_n)}{\Delta_n^{\ 2}}S_{n+1}^{(1)} + \frac{\xi(\xi-\Delta_n)^2}{\Delta_n^{\ 2}}S_n^{(1)}\,,\ \omega_n \le \omega \le \omega_{n+1} \quad (4)$$

where $\Delta_n = \omega_{n+1} - \omega_n$ and $\xi = \omega - \omega_n$. The function and its derivative value at $\omega_n$ respectively are

$$S_n = S(\omega_n)\,,\ S_n^{(1)} = \left.\frac{\partial}{\partial\omega}S(\omega)\right|_{\omega=\omega_n}. \quad (5)$$

### B. Evaluating $S(\omega)$ and $S^{(1)}(\omega)$ with a New Complex Integral Path

$S(\omega)$ and $S^{(1)}(\omega)$ are both in the form of GSI that is well known to be time consuming. In addition, the integrand of $S^{(1)}(\omega)$ appears singular when $\lambda = k_0$ if the integration path is along the real axis. That is due to the fact that the term $u_0^{-1}$ exists after $F(\omega,\lambda)$ is differentiated with respect to $\omega$. Let $S(\omega)$ be an example, the integral can be decomposed to be

$$S(\omega) = \left(\int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4}\right)F(\omega,\lambda)J_1(\lambda r)\mathrm{d}\lambda\,, \quad (6)$$

where the deformed complex path, as shown in Fig. 1, is

$C_1: \lambda = \xi - \mathrm{j}\frac{1}{r}\sin\frac{\xi}{2k_0}\pi,\ 0 \le \xi \le 2k_0; C_2: \lambda = \xi, 2k_0 \le \xi \le T;$

$C_3: \lambda = T + \mathrm{j}\xi, C_4: \lambda = T - \mathrm{j}\xi, 0 \le \xi < \infty$.

$T$ is a constant which can make $J_1$ be approximated by cosine function. With the path $C_1$, the singularity of $u_0^{-1}$ has been completely overcome. The $1/r$ appearing in the imaginary part is the precaution to avoid Bessel function $J_1$

going infinity. The path $C_3$ and $C_4$ will make the integral convergent very quickly [1].

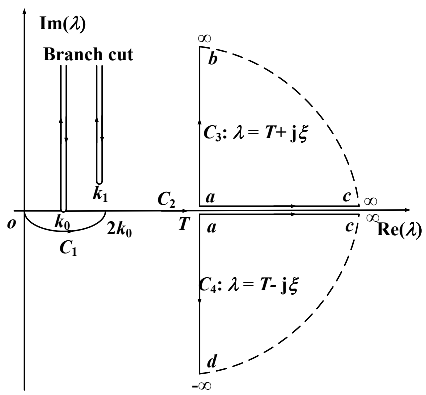


Fig. 1. Deformed integral path for evaluating the $S(\omega)$.

### C. Adaptive Sampling Strategy

The maximal frequency for evaluating lightning electromagnetic field is 10 MHz, so it is crucial to reduce the sampling frequency point. Besides, the real and imaginary part of $S(\omega)$ show a oscillatory property, which makes the Hermite model requires more sampling points. In lightning community, the Cooray-Rubinstein (CR) formula [2] has been widely utilized to evaluating $E_r(\omega)$ approximately to avoid the GSI computation. The expression of the CR's formula is,

$$E_{r,CR}(\omega) = E_{r,ideal}(\omega) - \sqrt{\frac{j\omega\mu_0}{j\omega\varepsilon_r\varepsilon_0 - \sigma}} H_{\varphi,ideal}(\omega, 0)\,, \tag{7}$$

where $H_{\varphi,ideal}(\omega,0)$ is the magnetic field on the earth surface assuming the ground to be a perfectly conducting plane. It is natural to assume that the zeros of the real and the imaginary part of $H_{\varphi,ideal}(\omega,0)$ could be served as the good initial guess frequency points for fitting $S(\omega)$. Using these frequency points, a Hermite model can be constructed using (4). Then, refining the intervals via adding the middle point of each interval, one can have a better approximated Hermite model. It is obvious that the above procedure can be conducted iteratively until the pre-defined accuracy is reached.

## III. NUMERICAL EXAMPLES

### A. Validation of $E_r$ in Time Domain

The lighting channel parameter can be found in [1]. To highlight the difference of $E_r$ using the CR formula in (7) and the GSI in (1), the ground conductivity is set to be $1\times10^{-4}$ S/m. The results using finite difference time domain (FDTD) are provided as well as references. The figures 2 and 3 are the $E_r$ in time domain. It can be found that the results using the CR formula have some errors for the poorly conducting ground [2].The results using (1) reach agreement with ones using FDTD satisfactorily.

### B. Comparison of the Computation Time

In figures 2 and 3, the horizontal electric fields are calculated at some frequency points which are determined by the approach described in Section II-C. The number of sampling points and the computing time are listed in Table I accordingly. As a contrast, if the Hermite interpolation approach is not used, $E_r$ at 1000 different frequency points over the lightning current frequency spectrum has to be calculated, and the computing time is given in Table I.

## IV. CONCLUSION

An adaptive Hermite interpolation in frequency domain is utilized for accelerating the process for obtaining the $E_r$ in time domain. In the extended version, more detailed comparison will be presented in terms of computational effort, computing time and accuracy.

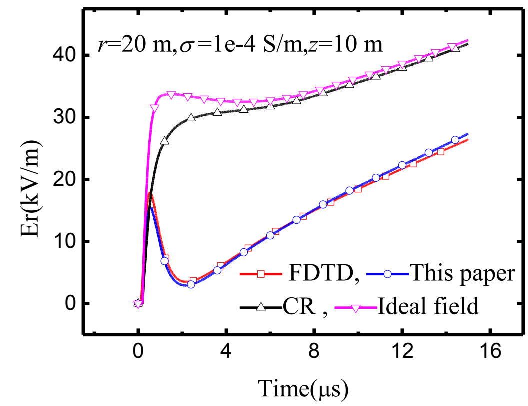


Fig. 2. Comparsion of $E_r$ in time domain when $r$ = 20 m

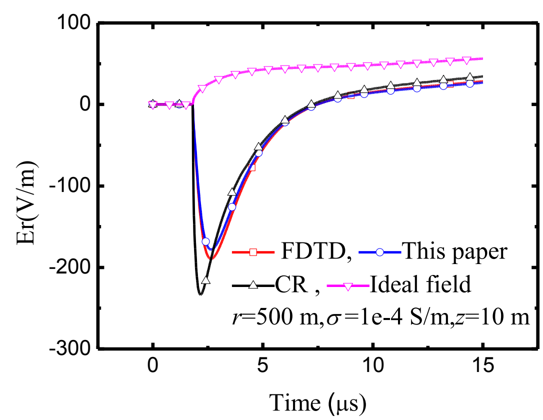


Fig. 3. Comparsion of $E_r$ in time domain when $r$ = 500 m.

TABLE I
COMPARISON OF THE COMPUTING TIME WITH AND WITHOUT INTERPOLATION

| $r$ /m | With Hermite Interp | | Without Hermite Interp | |
|---|---|---|---|---|
| | Numbers | Time /s | Numbers | Time /s |
| 20 | 36 | 2.75 | 1000 | 55.25 |
| 200 | 115 | 8.45 | 1000 | 67.82 |
| 500 | 210 | 19.81 | 1000 | 103.97 |

## V. REFERENCES


[1] Jun Zou, Boyuan Zhang, Xuelong Du et. al., “High-Efficient Evaluation of the Lightning Electromagnetic Radiation over a Horizontally Multilayered Conducting Ground With a New Complex Integration Path”, *IEEE Trans. on EMC*, vol. 56, no. 3, Jun. 2014, pp. 659-667.

[2] V. Cooray, "Horizontal electric field above and underground produced by lightning flashes", *IEEE Trans. on Elect. Compat.*, vol.52, no.4, pp.936-943, Nov. 2010.